ECE 307 – Techniques for Engineering Decisions

17. Value-at-Risk or *VaR*

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INTRODUCTION TO FUTURES

- **Commodity traders trade important commodities such as foodstuff, livestock, metals, oils, and electricity using financial instruments known as *forward contracts***
- **Standardized forward contracts are known as futures**

INTRODUCTION TO FUTURES

- Futures have finite lives and are primarily used to hedge commodity price-fluctuation risks or to take advantage of price movements, rather than for the purchase or sale of the actual cash commodity
- The buyer of the futures contract agrees on a fixed purchase price to buy the underlying commodity

INTRODUCTION TO FUTURES

from the seller upon the expiration of the contract; the seller of the futures contract agrees to sell the underlying commodity to the buyer at expiration at the fixed delivery price

- As time passes, the contract's price changes with respect to the specified price at which the trade was initiated
- This creates profits or losses for each trading side

INTRODUCTION TO FUTURES

- The word "contract" is used because a futures contract requires delivery of the commodity in a stated month in the future unless the contract is liquidated before it expires
- However, in most cases, delivery never takes place
- Instead, both the buyer and the seller, usually liquidate their positions before the contract expires; the buyer sells his futures and the seller buys futures and thus all the transactions are *financial* rather than *physical*

COMMODITY PORTFOLIOS

- **Traders usually hold portfolios of commodities: each portfolio comprises a collection of different commodities, each bought at a certain price/time, and comes with distinct terms and conditions**
- **Such holding is done in order to diversify the portfolio and thereby mitigate the overall *risk***
- **The value of a portfolio, at any given point in time, is determined by the sum of the individual values of each of the commodities in the 'basket'**

IMPORTANCE OF DIVERSIFYING THROUGH PORTFOLIO HOLDING

- **A well balanced and diversified portfolio provides benefits to the holder by lowering the overall risk**

- **The key reason is because market or other economic conditions that cause one futures contract to perform very well may often cause a different contract to perform rather poorly**

IMPORTANCE OF DIVERSIFYING THROUGH PORTFOLIO HOLDING

- Speculators and hedgers hold diversified positions so as not to have "all eggs in one basket"

- While diversification is not a guarantee against loss, it is an effective strategy to help manage the risk faced by the holder

PORTFOLIO ANALYSIS

- We consider a portfolio of investments, say securities, and we analyze and quantify the risk
- We first define the notation and the key metric
- We go over the steps for the determination of the *VaR* metric for a general case

MARKET UNCERTAINTIES

- We consider the purchase of a portfolio $\underset{\sim}{P}$ at time $t = 0$ for the overall price p_0
- The value of the portfolio varies over time and we denote its value at an arbitrary time t by p_t
- This portfolio is exposed to the various sources of uncertainty to which the market for each component or commodity is subjected and consequently its value fluctuates as a result of the impacts of the various sources of uncertainty

PERFORMANCE PREDICTION

- On any given trading day $t = T$, the fixed portfolio may either incur a loss or a gain or remain unchanged with respect to its value at $t = T - 1$
- We wish to determine what the *worst performance* of the portfolio may be from the day $t = T - 1$ to the day $t = T$ and how to systematically measure the performance over any two consecutive days

PERFORMANCE PREDICTION

- On day $t = T$, we cannot lose more than the overall value p_T of the portfolio and this statement holds true with a probability of 1
- In other words, with a probability of 1, the loss must be less than or equal to p_T

PORTFOLIO VALUE AND RETURNS

- We evaluate the change δ_T in the portfolio closing value p_t from day $t = T - 1$ to day $t = T$ as:

$$\delta_T = p_T - p_{T-1}$$

- We define the rate of return r_t of the portfolio from day $t = T - 1$ to day $t = T$ in terms of δ_T to be

$$r_T = \frac{\delta_T}{p_{T-1}}$$

PORTFOLIO VALUE AND RETURNS

- The value of r_t for each observation is the change in the portfolio value from day $t = T - 1$ to day $t = T$ *normalized by the protfolio value on day* $T - 1$
- The value of r_t must lie in the interval $[-1, \infty)$
- A non-positive value of r_t indicates a loss in the portfolio value from day $t = T - 1$ to day $t = T$

DATA COLLECTION

- Suppose that we have the set of r_t data for a good number of days
- We analyze the set of r_t data obtained from sampling from a population the realizations of the portfolio price random variable $\underset{\sim}{P}$ with the past values

$$\{ p_0 , p_1, \ldots , p_{T-1}, p_T , \ldots \}$$

- We use the *r.v.* $\underset{\sim}{P}$ to define the two *r.v.*s $\underset{\sim}{\Delta}$ and $\underset{\sim}{R}$
- The sample values of $\underset{\sim}{R}$ are $\{ r_1, r_2, \ldots , r_{T-1}, r_T , \ldots \}$

DATA COLLECTION

date	*close price*	*loss/gain*	*percent loss/gain*
3/5/2007	$42.15	-$0.33	-0.78%
3/2/2007	$42.48	-$0.65	-1.51%
3/1/2007	$43.13	-$0.20	-0.46%
2/28/2007	$43.33	$0.14	0.32%
2/27/2007	$43.19	-$1.85	-4.11%
.	.	.	.
.	.	.	.
.	.	.	.
3/18/1999	$105.12	$2.00	1.94%
3/17/1999	$103.12	-$0.75	-0.72%
3/16/1999	$103.87	$0.87	0.84%
3/15/1999	$103.00	$2.88	2.88%
3/12/1999	$100.12	-$2.50	-2.44%
3/11/1999	$102.62	$0.50	0.49%

$\underset{\sim}{P}$ $\underset{\sim}{R}$ $\underset{\sim}{\Delta}$

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$r_{3/1/2007}$

$\delta_{3/1/2007}$

$p_{3/1/2007}$

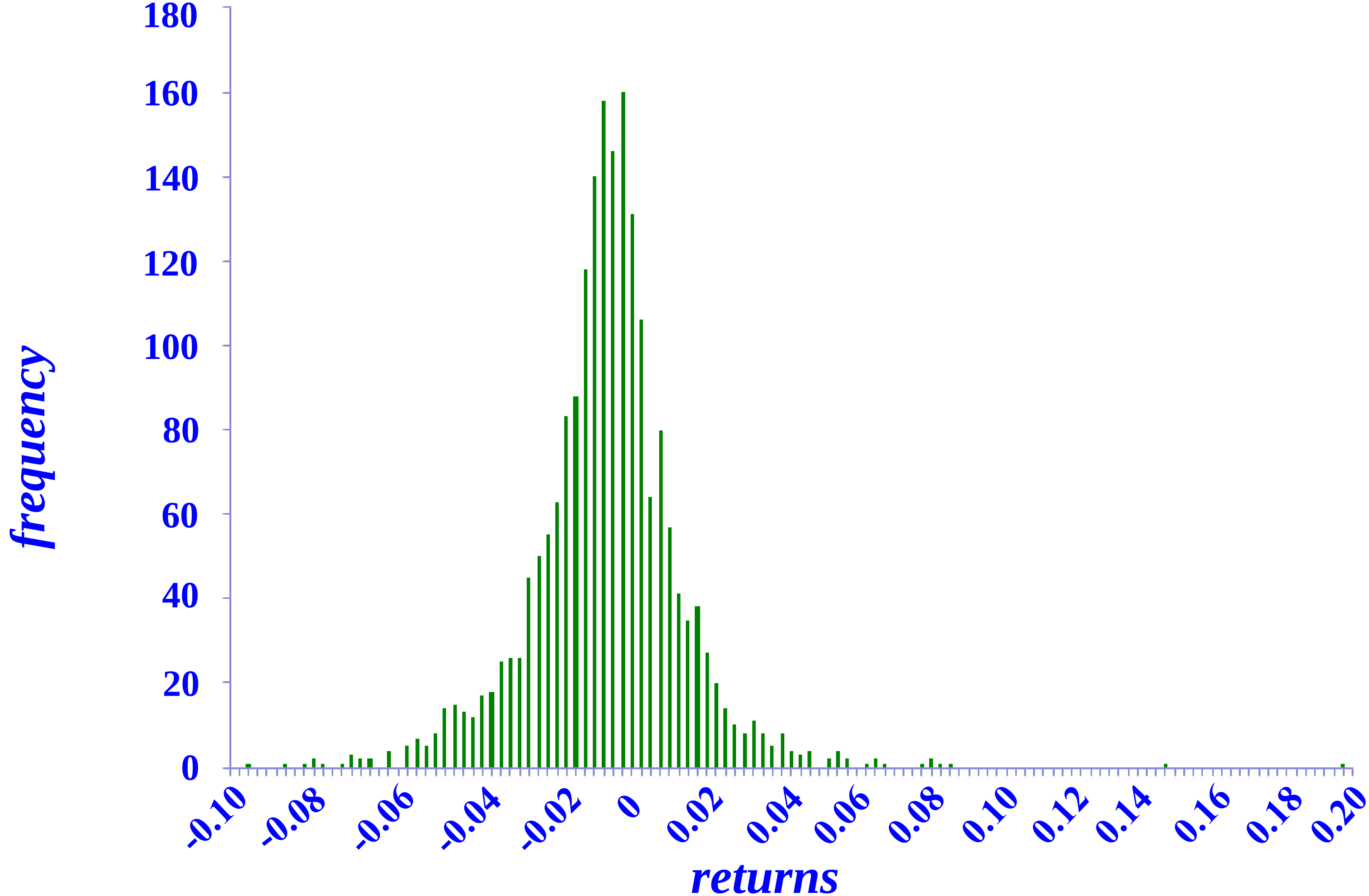
DATA COLLECTION

- We can use the historical values of $\underset{\sim}{R}$ to construct a probability distribution function
- The first step is to determine the frequency of $\underset{\sim}{R}$ taking on values in certain intervals; for this purpose, we discretize $\underset{\sim}{R}$ and define 'buckets' in which we drop the realized values of $\underset{\sim}{R}$
- The number of values in each bucket is used to compute the frequency of $\underset{\sim}{R}$ taking on a value in that bucket

'BUCKETS' AND FREQUENCY

buckets	*frequency*
-10.00 %	*0*
-9.75 %	*0*
-9.50 %	1
-9.25 %	*0*
.	.
.	.
-0.50 %	118
-0.25 %	140
0.00 %	158
0.25 %	146
0.50 %	160
.	.
.	.
19.25 %	*0*
19.50 %	*0*
19.75 %	1
20.00 %	*0*

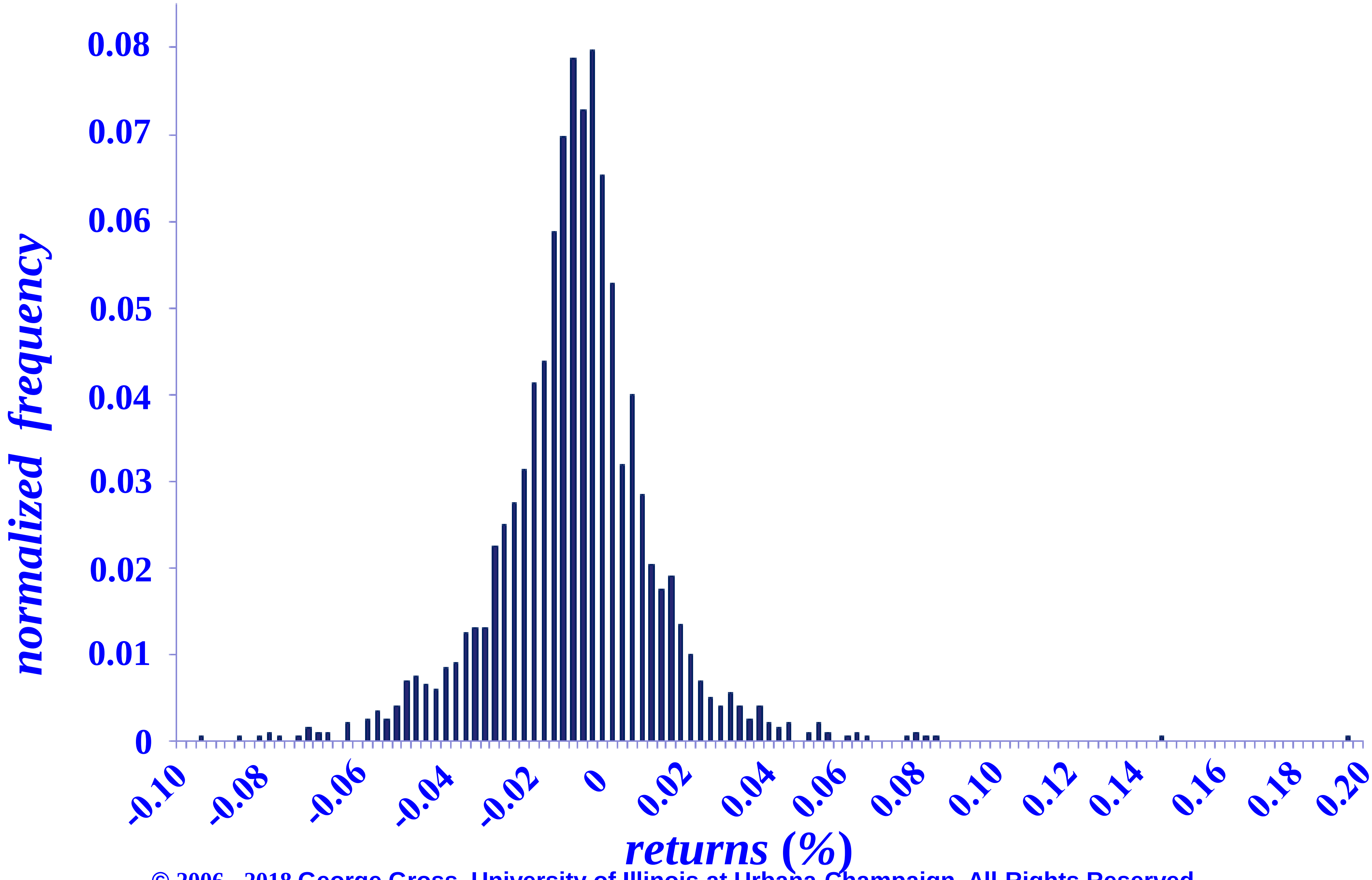
FREQUENCY VS. RETURNS DISTRIBUTION



NORMALIZATION

- **We first normalize these frequencies using the total number of observations and interpret the normalized quantities as the values of a discrete probability mass distribution function**
- **We then construct the corresponding *c.d.f. approxi-mation* from the data, and interpret the results in terms of the returns realized for the portfolio**

NORMALIZED FREQUENCY DISTRIBUTION



CUMULATIVE DISTRIBUTION FUNCTION (*c.d.f.*) of $\underset{\sim}{R}$

this c.d.f. gives the

cumulative values of

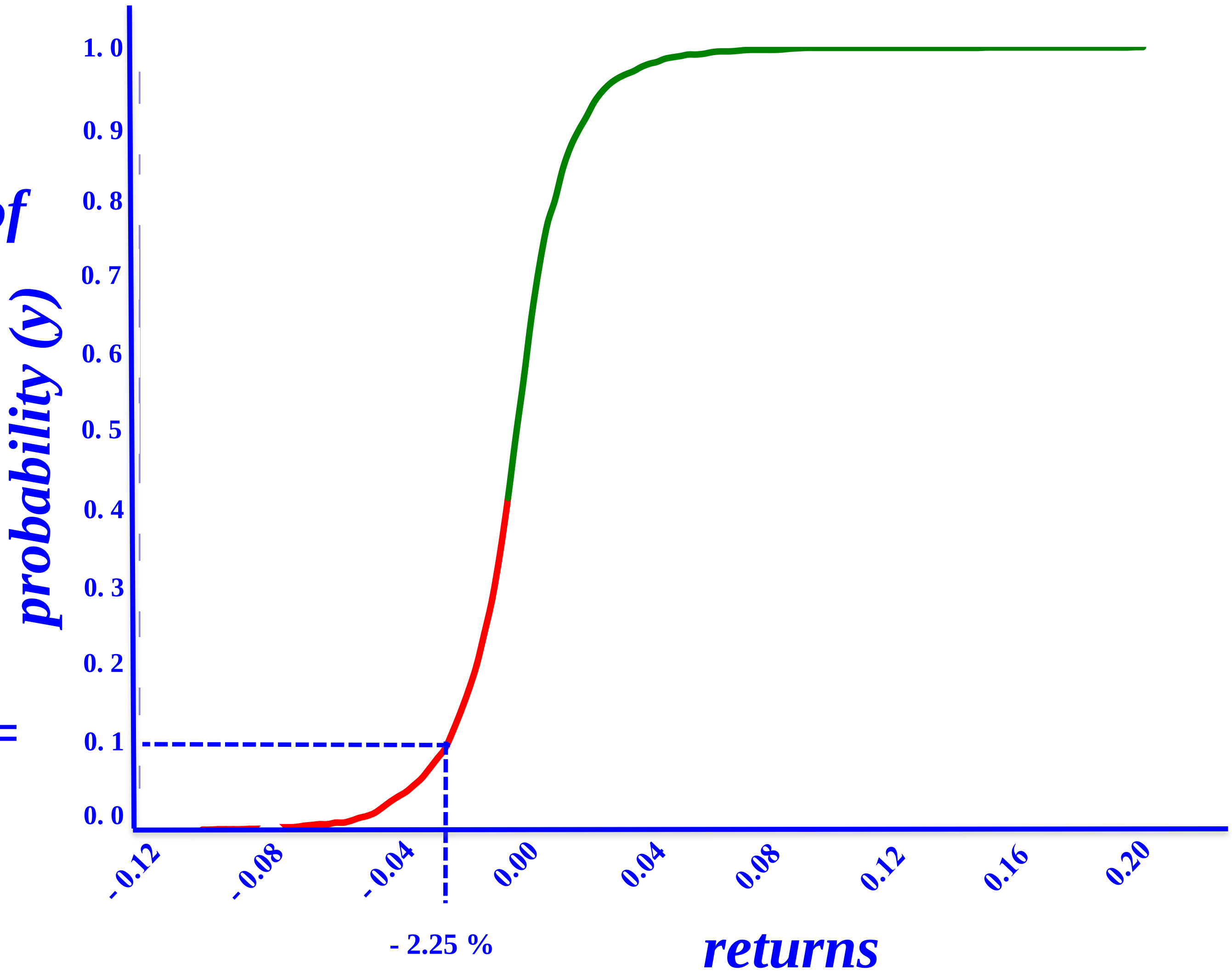
probability

$P\{ \underset{\sim}{R} \leq r \} = y$

For example:

$P\{ \underset{\sim}{R} \leq -2.25\ \%\} =$

0.1



INTERPRETING THE *c.d.f.*

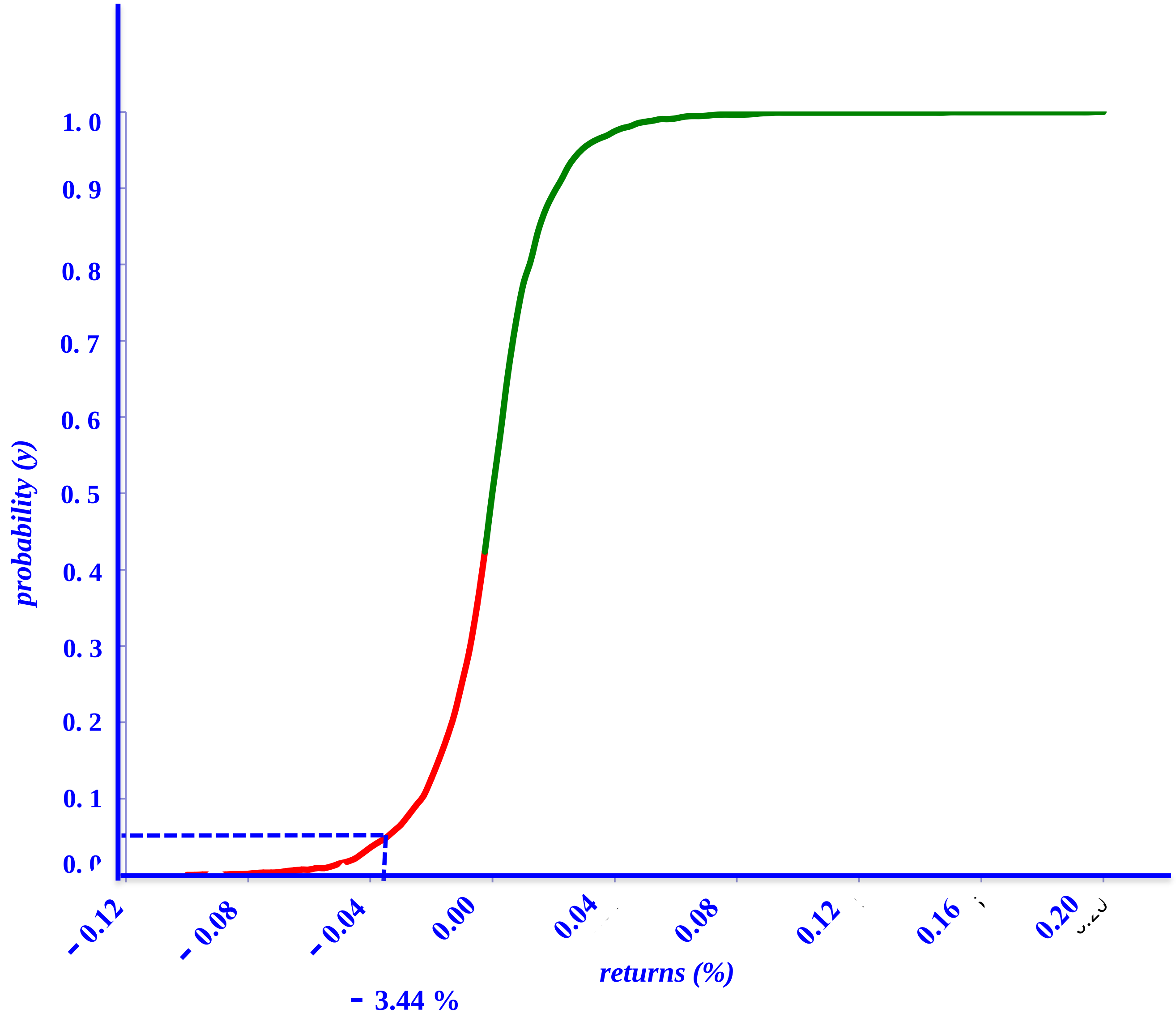
- We consider the data set to be representative of the distribution of the population of the outcomes collected from the past trading days of interest
- We focused on the example "the probability that $\underset{\sim}{R}$ is less than or equal to $-2.25\ \%$ is 0.1"
- We may view the complement of the probability value (0.1) as a "confidence level" with probability 0.9 and so we restate the above as "*with a confidence level of* 0.9, $\underset{\sim}{R}$ *will exceed* $-\ 2.25\ \%$"

UNDERSTANDING THE *c.d.f.*

- In general, for confidence level $(1 - y)$, the information provided by the *c.d.f.* approximation allows us to determine the value r that $\underset{\sim}{R}$ exceeds $100\,(1 - y)\,\%$ of the days based on the collected observations in the data set
- For example, the *c.d.f.* approximation implies that with a 0.95 confidence level, $\underset{\sim}{R}$ exceeds $-3.44\,\%$
- We can interpret this statement to mean that with a confidence level of 0.95 we don't expect to lose more than $3.44\,\%$ of the previous day value in the worst case over any two consecutive days

c.d.f. OF $\underset{\sim}{R}$

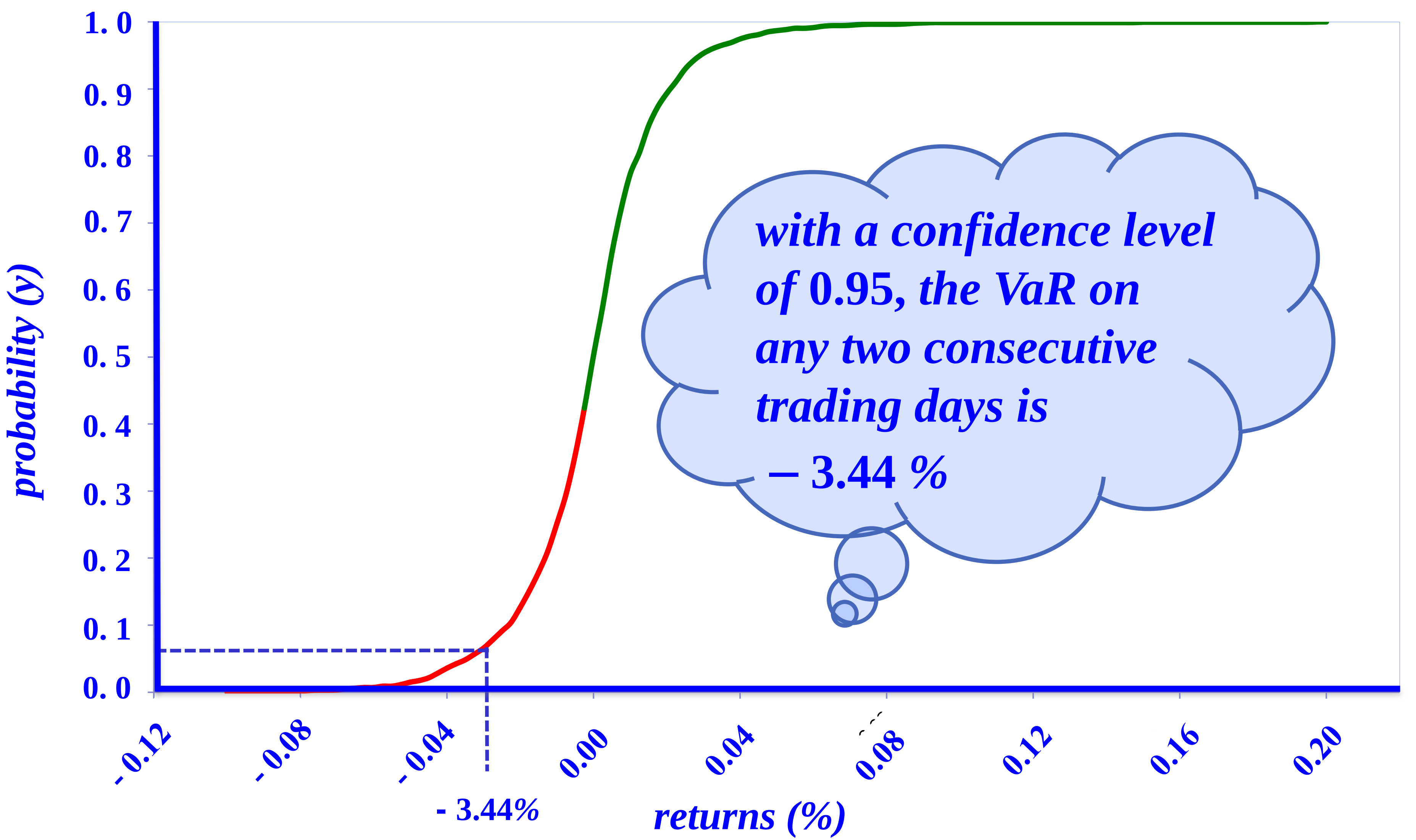
with a confidence level of 95 % we don't expect to lose more than 3.44 % in the worst case



VALUE – AT – RISK (VaR)

- **Terminology: "with a confidence level of 0.95, the *VaR* is – 3.44 %" means that with a 0.95 percent confidence level, the return over any two consecutive days cannot be below – 3.44 %**
- **A negative *VaR*, say $v < 0$, means that the *losses* on any one day cannot exceed $-v$ %**
- ***VaR* is a measure of the return which is exceeded based on the observations available for the past time period, with the specified confidence level**

CUMULATIVE DISTRIBUTION FUNCTION (*c.d.f.*)



VALUE – AT – RISK (VaR)

- ***VaR*** **is usually expressed as a percentage value of the portfolio**
- ***VaR*** **answers the fundamental question facing a risk manager – on any given day, how much can I lose at the specified confidence level?**
- **The entire procedure may be extended to determine returns over any time period (e.g., two days, a week, or a month, etc.) and *VaR* can therefore be calculated for any such period**

VALUE – AT – RISK (VaR)

- *VaR* is commonly used by banks, security firms and companies that are involved in trading energy and other commodities

- *VaR* is a measure of risk as it happens and provides an important metric for firms that make trading or hedging decisions

A PRACTICAL ASSIGNMENT

- Pick any 5 stocks and construct a 1,000-share portfolio equally weighted (200 shares each) from each of the 5 stocks
- Obtain historic stock price data starting from January 1 , 2002 (http://finance.yahoo.com)
- Calculate $\underset{\sim}{\Delta}$ and $\underset{\sim}{R}$ for each $\underset{\sim}{P}$ observation: assume that all dividends are reinvested to purchase more stock (fractional amounts, if necessary)

ASSIGNMENT

- **Plot the normalized frequency distribution and the *c.d.f.* for the data collected**
- **Compute the *VaR* for the confidence levels 95 % and 99 %**
- **Interpret what these values mean specific to your particular portfolio**